HAP Code Retreat: Simulating Blazars with a two-zone-hybrid and a spatially resolved SSC model

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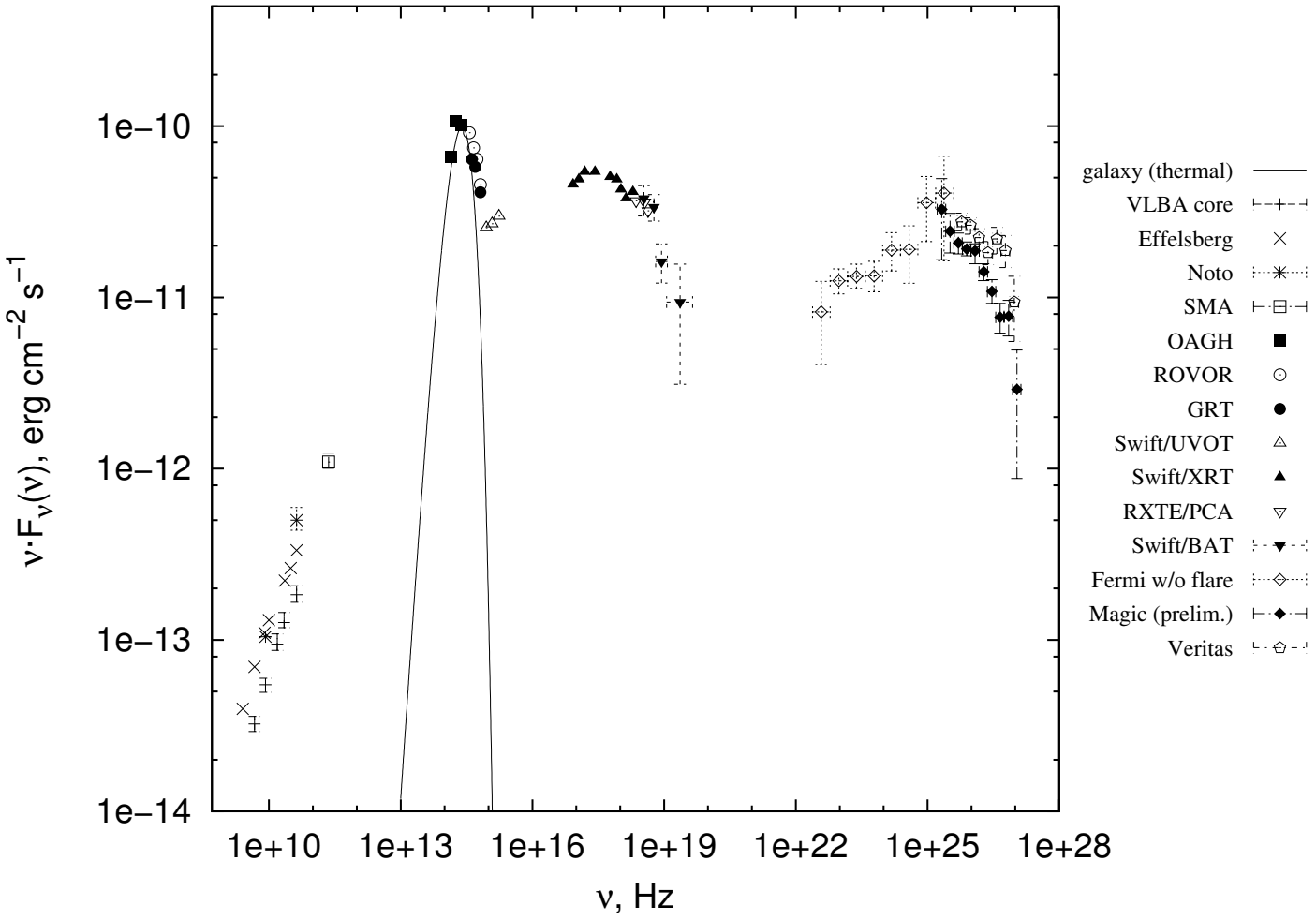
May 5, 2020

Outline

1. Two-zone-hybrid model (COJONES)
2. Spatially resolved SSC model (UNICORN)
   - Motivation
   - Geometry
   - Acceleration
   - Radiation
   - Numerics
   - Results
   - Outlook

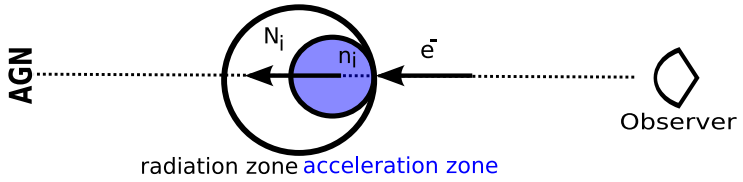
The challenge

Model highly relativistic regions (so called blobs), moving along a jet, accelerating electrons (and possibly protons) to Lorentz factors of $\sim 10^7$, whose radiated spectrum looks approximately like this (example of *Markarian501*):



COJONES - SSC part

Assume spherical emitting and acceleration region containing isotropic particle distributions and randomly orientated magnetic field in a nested setup:



see Weidinger et al. 2010 for details

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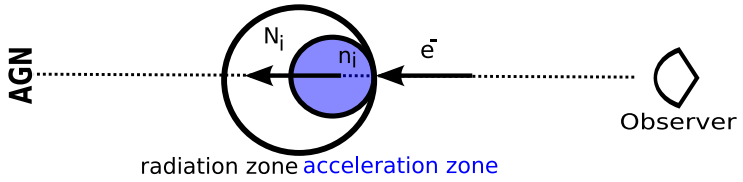

Kinetic equation: acceleration zone

$$\partial_t n_e = \partial_\gamma \left[(\beta_s \gamma^2 - t_{\mathrm{acc}}^{-1} \gamma) \cdot n_e \right] + \partial_\gamma \left[[(a+2) t_{\mathrm{acc}}]^{-1} \gamma^2 \partial_\gamma n_e \right] + Q_0 - t_{\mathrm{esc}}^{-1} n_e$$

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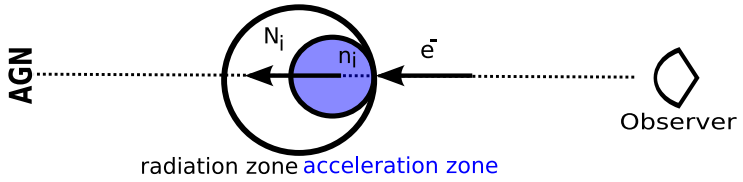

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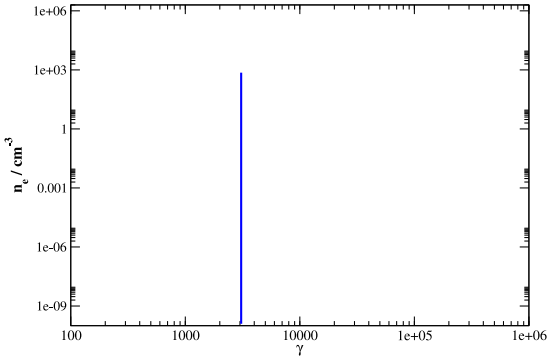

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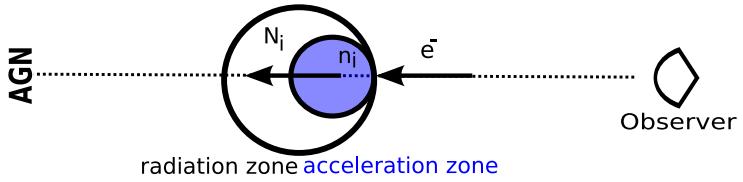

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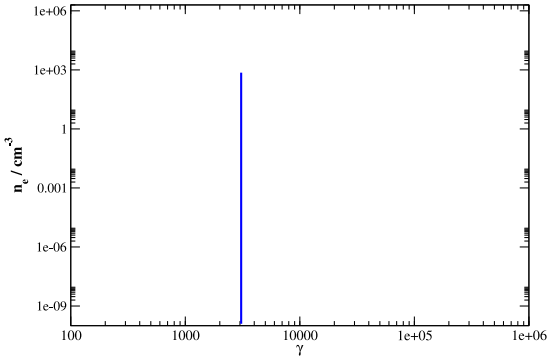

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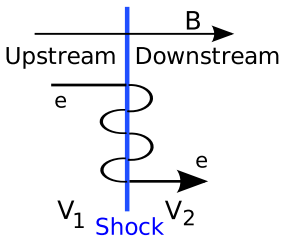

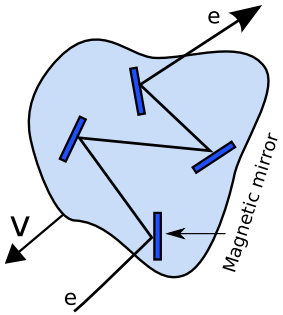

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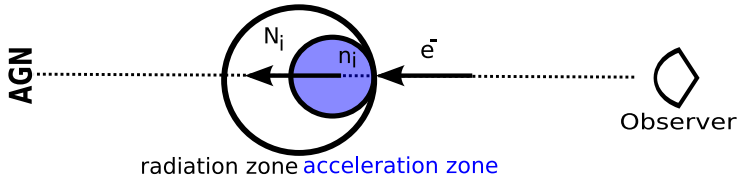

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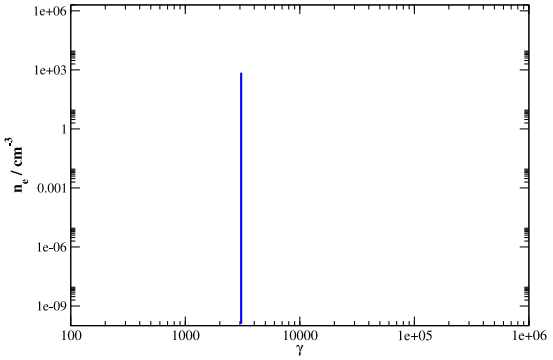

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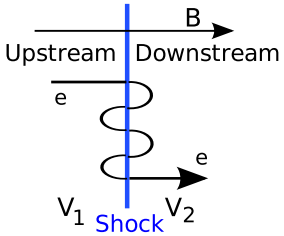

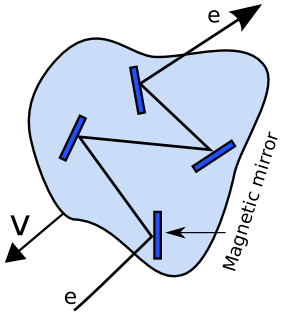

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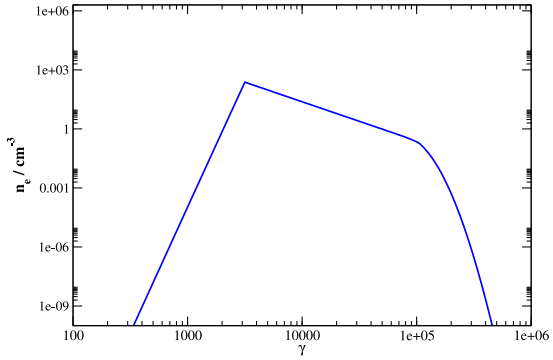

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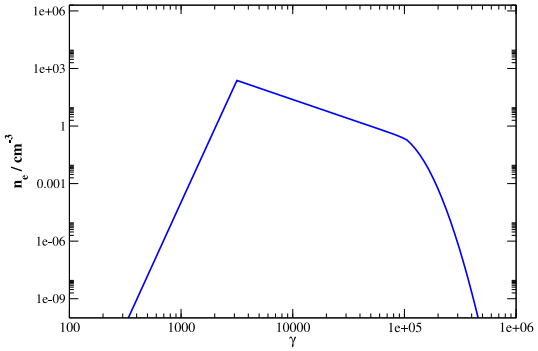

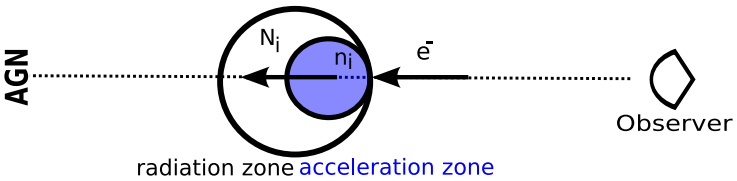

Kinetic equation: radiation zone

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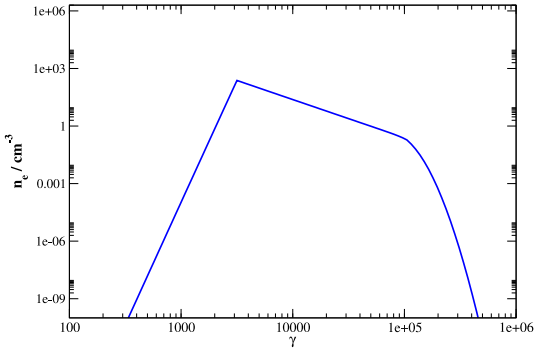

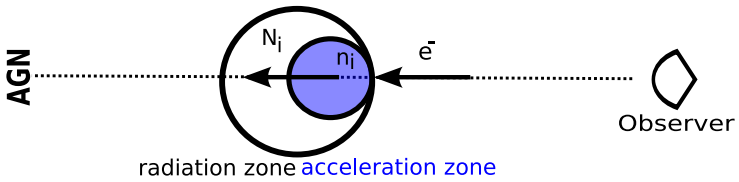

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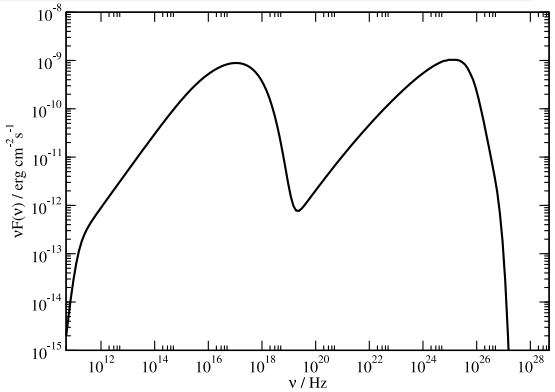
Photon distribution

$$\partial_t N_{\mathrm{ph}} = R_{\mathrm{syn}} + R_{\mathrm{IC}} - c\alpha_{\mathrm{SSA}} N_{\mathrm{ph}} - t_{\mathrm{esc,ph}}^{-1} N_{\mathrm{ph}}$$

Selfconsistent SSC limit

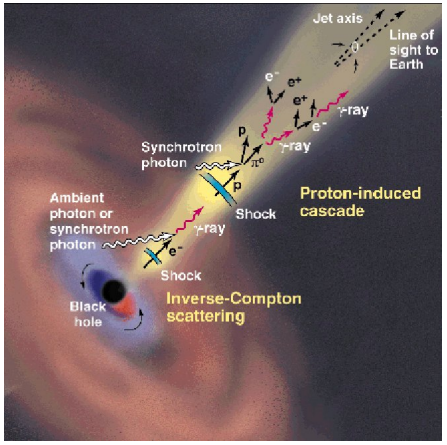
see Weidinger and Spanier 2010 for details



COJONES - add photo hadronics

Photo-Meson production:

- $p + \gamma \rightarrow p + n_0\pi^0 + n_+\pi^+ + n_-\pi^-$
- $\pi^\pm \rightarrow \mu^\pm + \nu_\mu/\bar{\nu}_\mu$
- $\pi^0 \rightarrow \gamma + \gamma$
- $\mu^\pm \rightarrow e^\pm + \nu_e/\bar{\nu}_e + \bar{\nu}_\mu/\nu_\mu$



Resulting γ-radiation above $\approx 10^{28}$ Hz will be opaque to

$$\gamma + \gamma \rightarrow e^+ + e^-$$

$\Rightarrow$ pair induced cascades and cascade radiation will emerge in jets with non-thermal p^+ present[1].

COJONES - add photo hadronics

Now there are 4 **non-linear coupled** equations in the radiation zone:

Kinetic equations: radiation zone

$$\partial_t N_{p^+} = \partial_\gamma \left[\beta_p \gamma^2 \cdot N_{p^+}\right] + b^3 t_{\mathrm{esc,p}}^{-1} n_{p^+} - t_{\mathrm{esc,p,N}}^{-1} N_{p^+}$$

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- Kelner Aharonian parameterization of the SOPHIA Monte Carlo results is used to calculate $Q_{p\gamma^-}$, $Q_{p\gamma^+}$, R_{π^0} $\Rightarrow$ unstable intermediates $(\pi^\pm, \pi^0, \mu^\pm)$ are not taken into account

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- Kelner Aharonian parameterization of the SOPHIA Monte Carlo results is used to calculate $Q_{p\gamma^-}$, $Q_{p\gamma^+}$, R_{π^0} $\Rightarrow$ unstable intermediates $(\pi^\pm, \pi^0, \mu^\pm)$ are not taken into account
- Cascades will emerge in the optically thick regime $> 10^{28}$ Hz

A spatially resolved model - UNICORN

Motivation

- acceleration due to Fermi I happens at finite size shock
  $\Rightarrow \quad R_{acc} \ll R_{rad}$
- acceleration efficiency should depend on distance to shock
- for energies with $t_{cool} < lct$ the blob simply isn't homogenous $\Rightarrow$ effect on global spectral energy density (SED)

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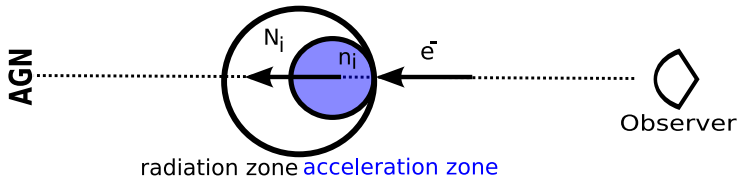
- time lags might be explained by light travel time between different components of the blob
- cooling and reacceleration of intermediates of e.g. photohadronic processes can be treated in detail

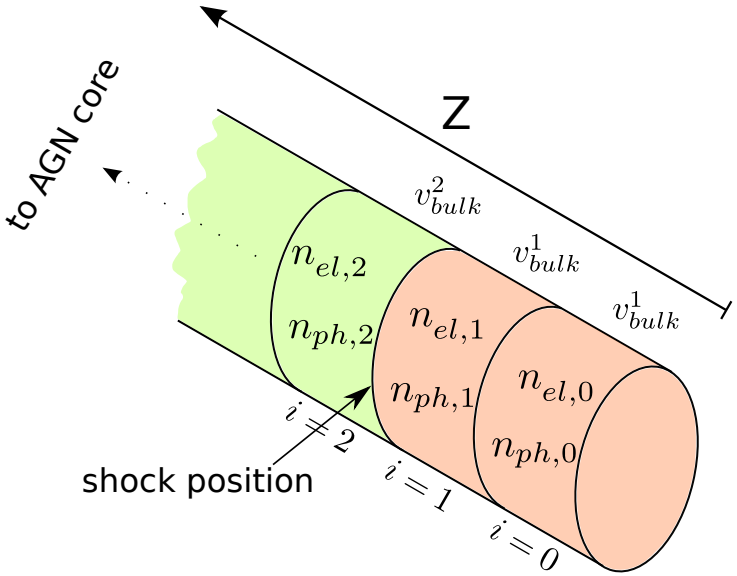
Back to SSC



expand two-zone-model to N-zone-model

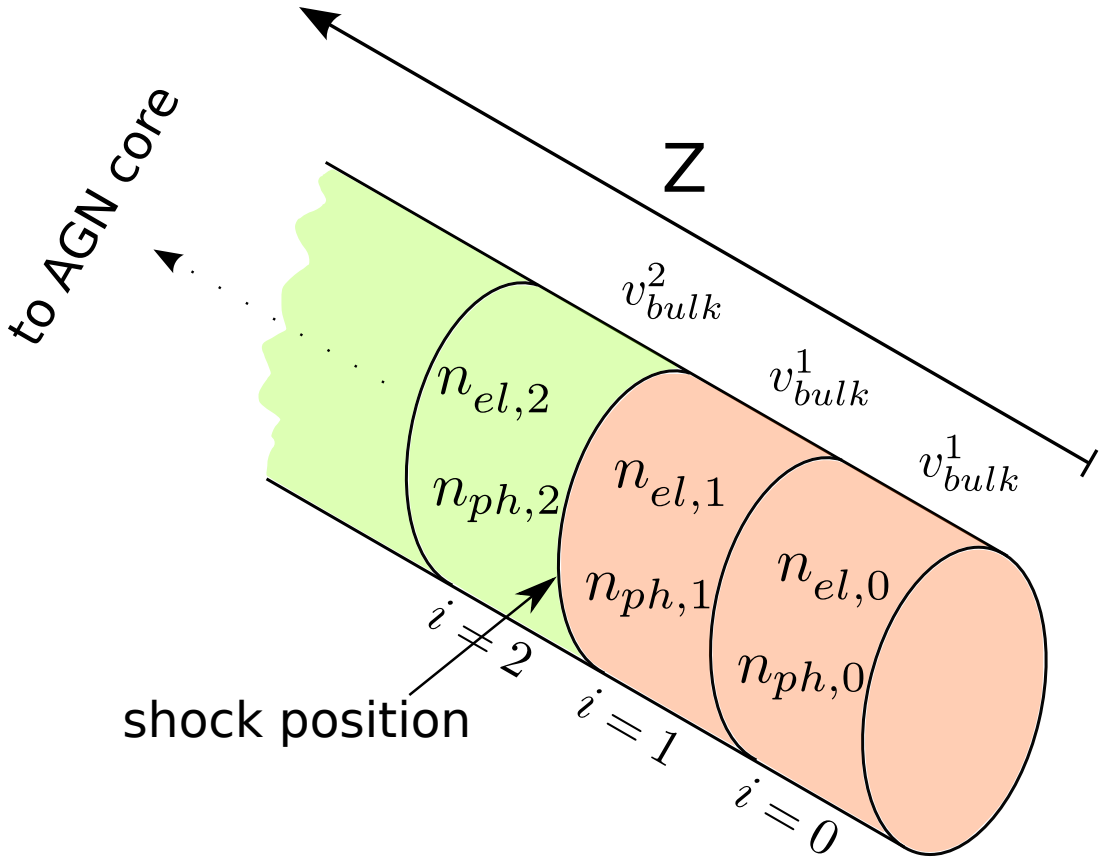
$\Downarrow \quad \Downarrow$

move from nested to aligned setup

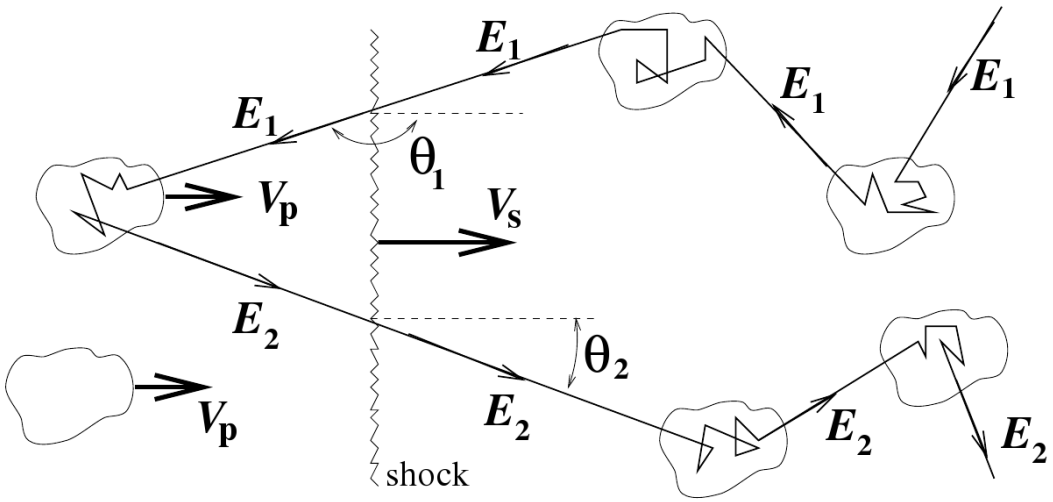


Model geometry

- devide simulation region into N slices in the direction parallel to shock normal
- each slice has local bulk speed, electron density, photon density (, magnetic field, radius)

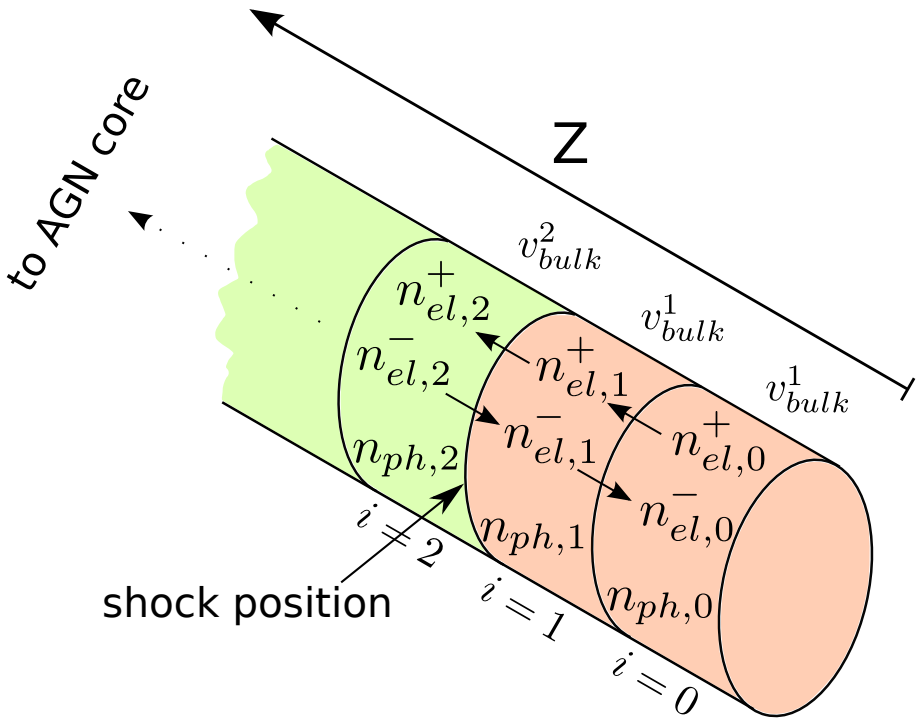


Acceleration - Fermi I



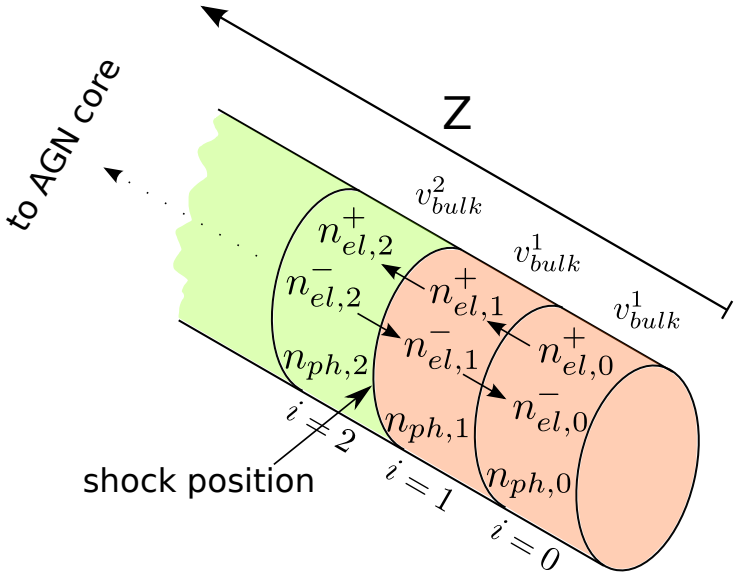
- multiple scattering on both sides of a shock
- scattering is elastic in plasma frame
- when particles cross the shock, the boost into the bulk frame distorts the isotropy of the target distribution
- hence head on collisions are more likely and acceleration becomes more efficient

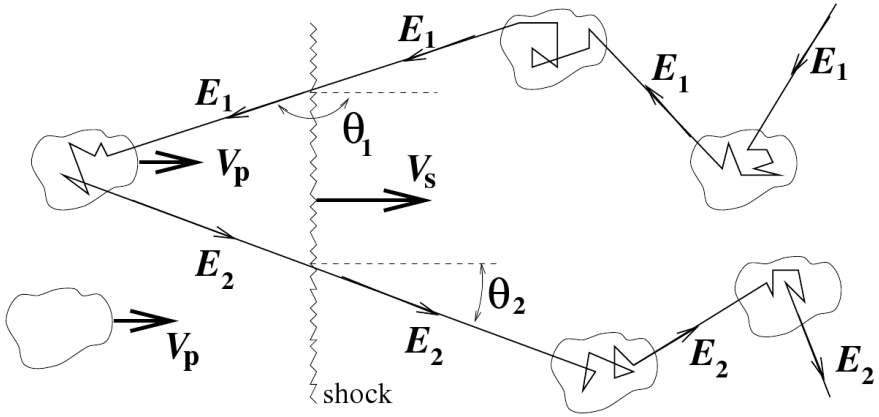
from geometry to acceleration



- split electron population into two half spheres; one moving downstream (n^+), the other moving upstream (n^-)
- connecting the slices via advection of electrons between them
- shock is represented by jump in bulk velocity u between neighboured zones

mimic Fermi-I





- in shock frame: $v^1_{bulk} = u_u = -V_S$, $v^2_{bulk} = u_d = V_P - V_S$, $R = \frac{u_u}{u_d}$
- relativistic treatment yields $V_P = \frac{V_S(R-1)}{R-V_S^2}$

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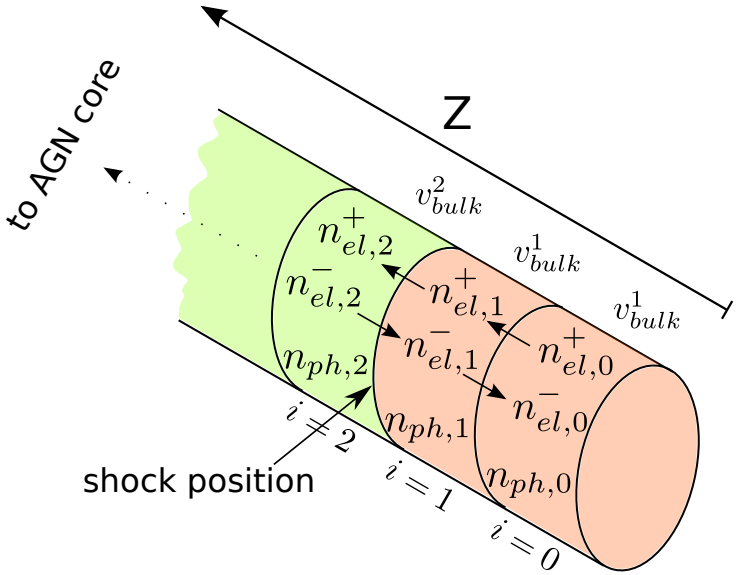

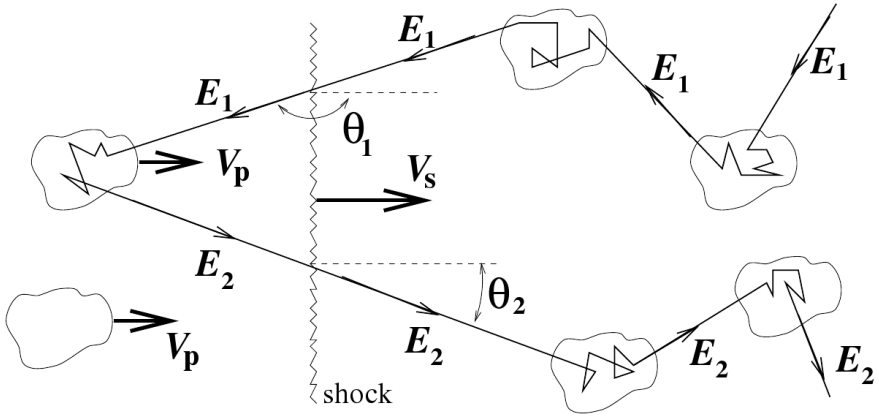

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- relativistic treatment yields $V_P = \frac{V_S(R-1)}{R-V_S^2}$
- scattering is controlled via the probability for an electron to change its propagation direction (including the boost into the new frame of reference)

Acceleration - Fermi II

- stochastic acceleration independent of presence of shock
- repeated scattering of relativistic particles leads to acceleration second order in bulk velocity
- included in model as diffusion in momentum space

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Fermi-II acceleration included in kinetic equation

$$\frac{\partial F}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 \cdot \left(\langle \dot{p}_{FII} \rangle \frac{\partial F}{\partial p} + \dot{p}_{cool} \cdot F\right)\right] + S(\mathrm{x}, \mathrm{p}, t)$$

$$\text{with } \langle \dot{p}_{FII} \rangle = p^2 \frac{V_A^2}{9\kappa_{\|}} = p^2 D$$

V_A ... Alfvén velocity; $\kappa_{\|}$ ... parallel diffusion coefficient

Radiation

time evolution of photon density

$$\frac{\partial N}{\partial t} = -c \cdot \kappa_{\nu,\mathrm{SSA}} \cdot N + \frac{4\pi}{h\nu} \cdot (\epsilon_{\nu,\mathrm{IC}} + \epsilon_{\nu,\mathrm{sync}})$$

- $\kappa_{\nu,\mathrm{SSA}}$ - Synchrotron Self Absorption coefficient

  calculated using the Melrose Approximation
- $\epsilon_{\nu,\mathrm{IC}}$ - changes due to invers compton scattering

  full integration of photon and electron density using the Klein-Nishina cross section
- $\epsilon_{\nu,\mathrm{sync}}$ - yields due to synchrotron radiation

  integration of electron density using the Melrose approximation for a single electron spectrum

see Richter and Spanier 2012 for details

Control flow

- update shock properties

electrons

- calculate the change in electron density for each slice due to convection from/to neighbouring regions and the source function
- compute backreaction of photon density (inverse compton) on electrons
- integrate the Vlasov-Equation in momentum space in each slice
- perform scattering of electrons from one half-space into the other due to pitch angle scattering

photons

- compute synchrotron power
- calculate synchrotron self absorption
- compute inverse compton gains and losses
- update photon density

Opencl implementations

numerical costs:

- due to statistical approach to Fermi I higher γ_{max}, hence smaller dt
- compute inverse compton scattering in each slice
  $\Rightarrow \mathcal{O}(N_z \cdot N_\nu \cdot (N_\gamma N_\nu))$

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update electrons	inverse compton scattering

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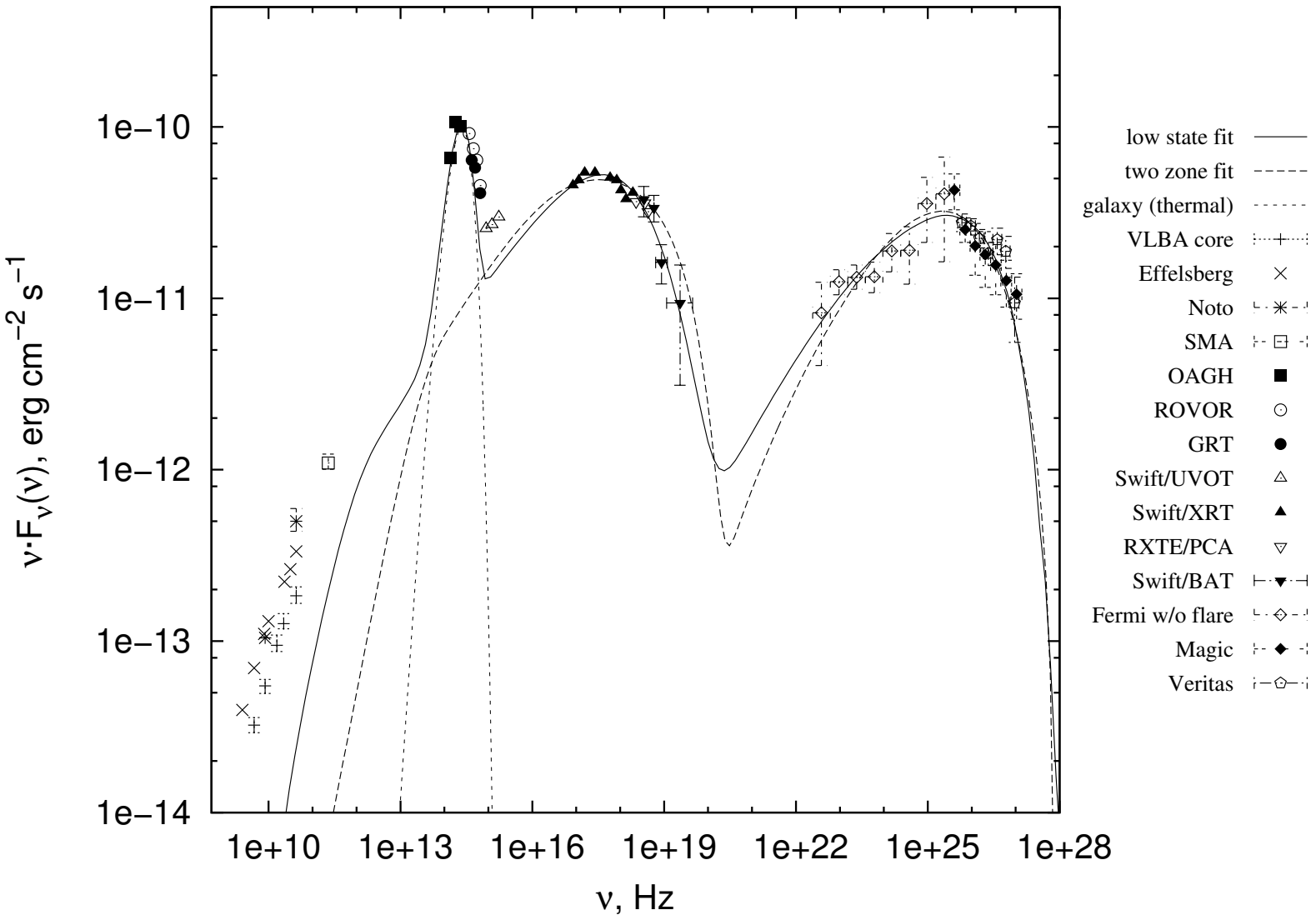
Results

Radio spectrum



Radio spectrum

synchrotron self absorption by a powerlaw distribution

equivalence of brightness and kinetic temperature leads to a flux

$$F_\nu \propto \nu^{\frac{5}{2}}$$

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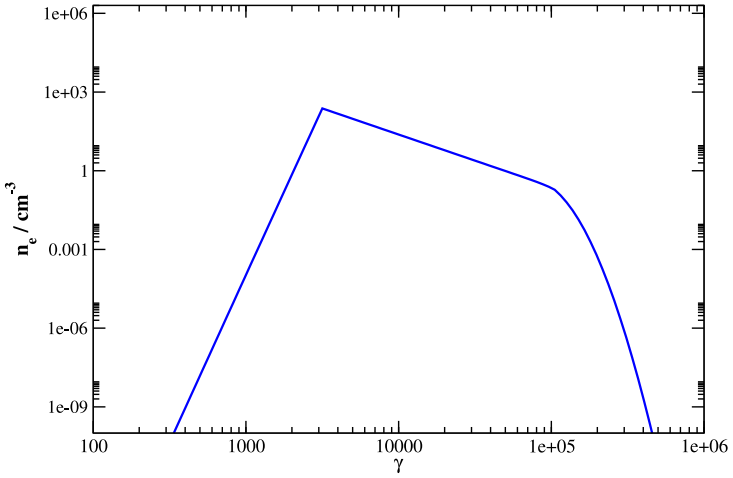

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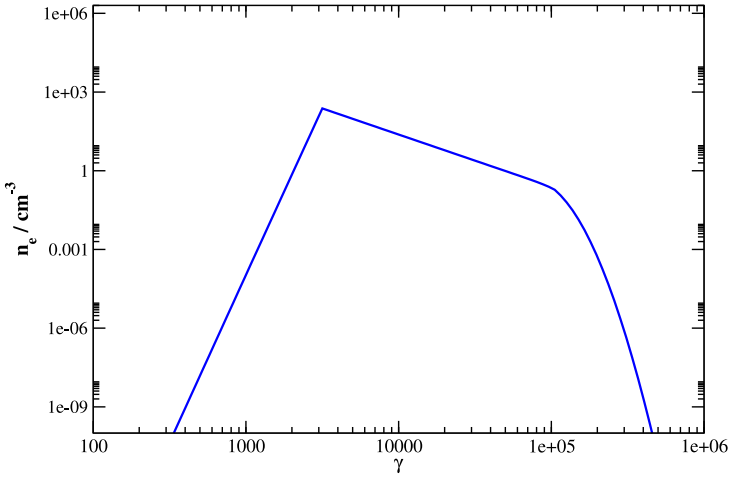
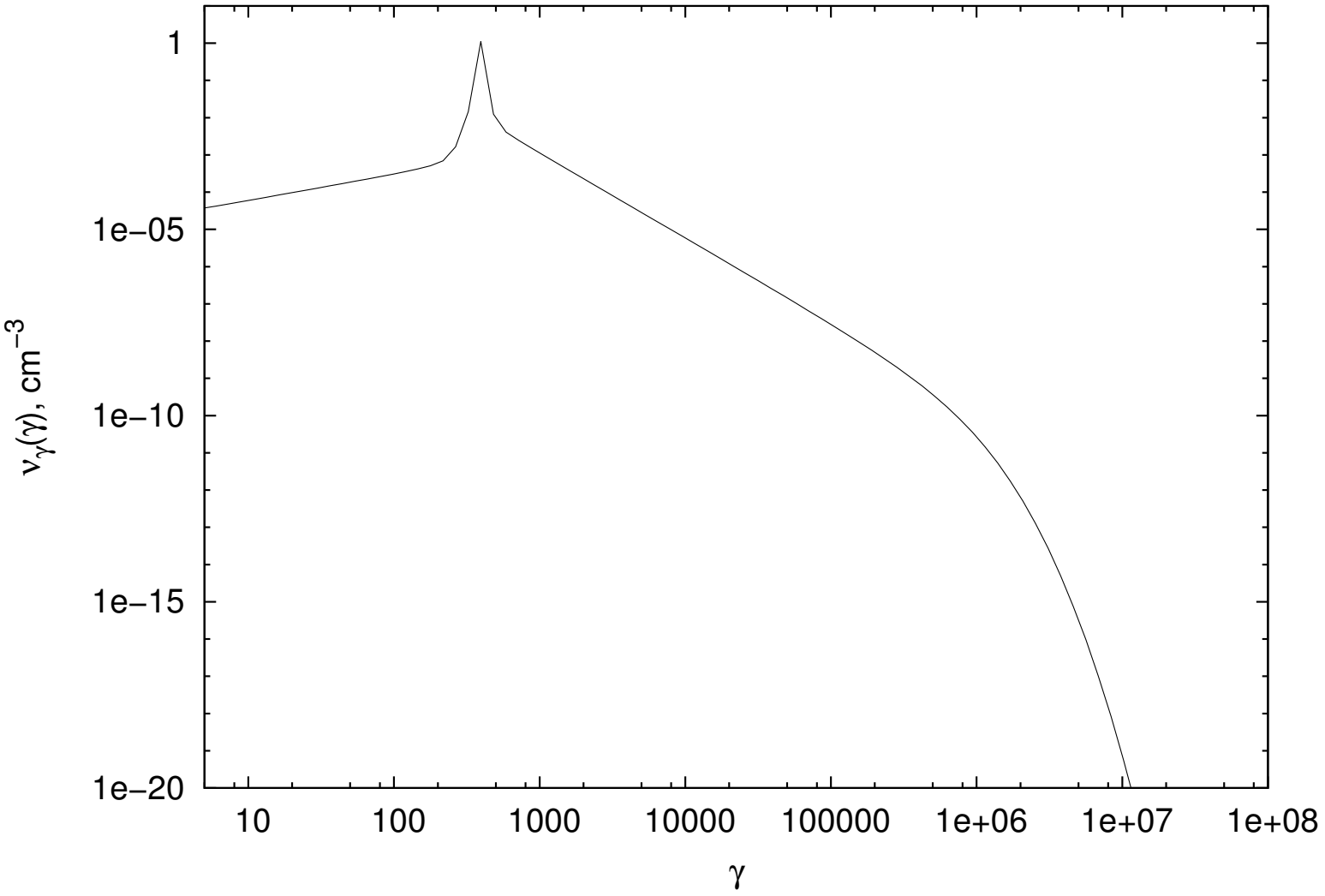


synchrotron radiation of a single electron

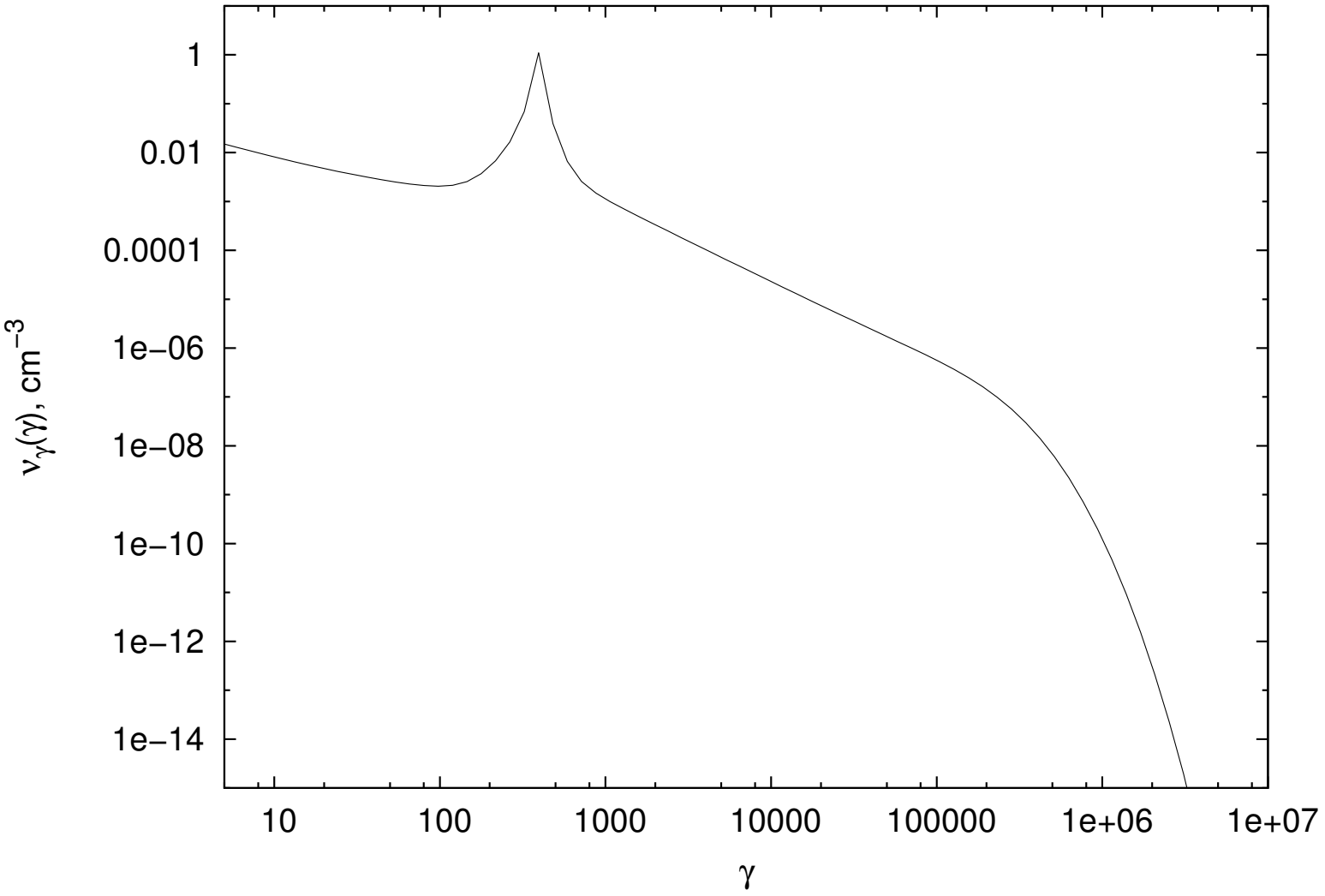
$$P_\nu(\gamma,\nu) = \frac{\sqrt{3}\ q^3 B}{m\ c^2} \cdot \frac{\nu}{\nu_c} \int_{\frac{\nu}{\nu_c}}^{\infty} \mathrm{d}\eta\ K_{\frac{5}{3}}(\eta) \overset{\text{small } \nu}{\propto} \nu^{\frac{1}{3}}$$

electron distribution development



Electron distribution at different distances to the shock.

electron distribution development



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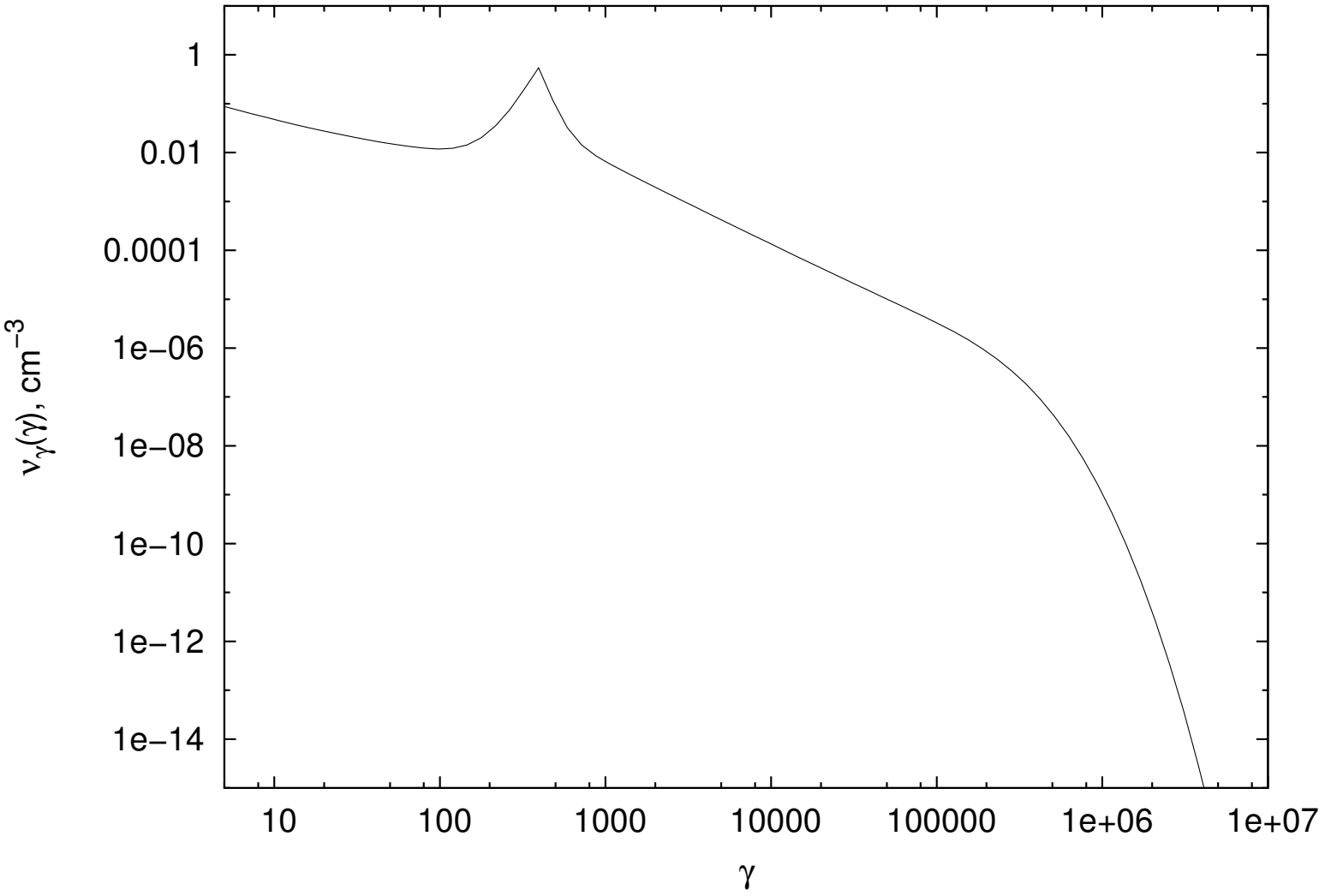

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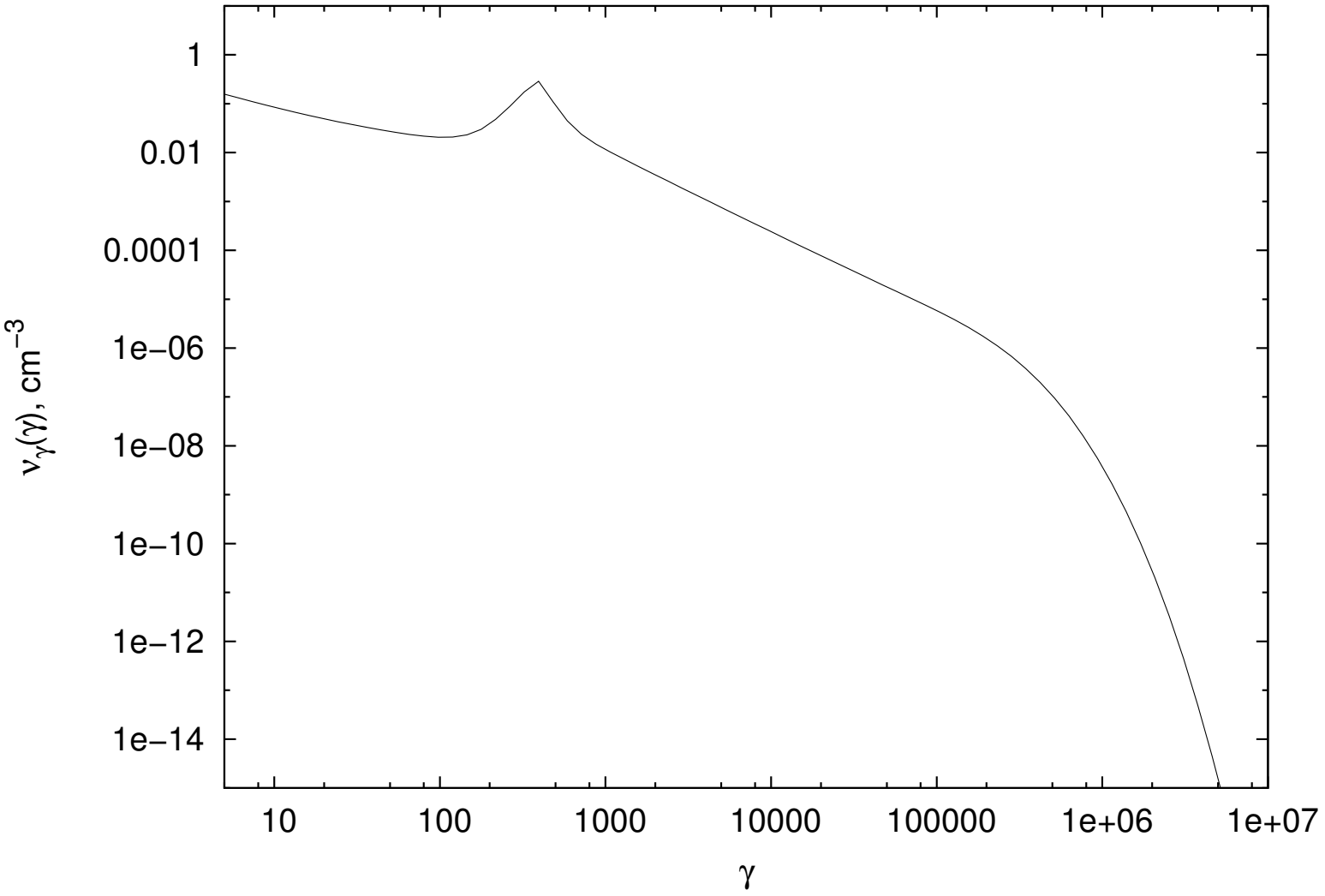

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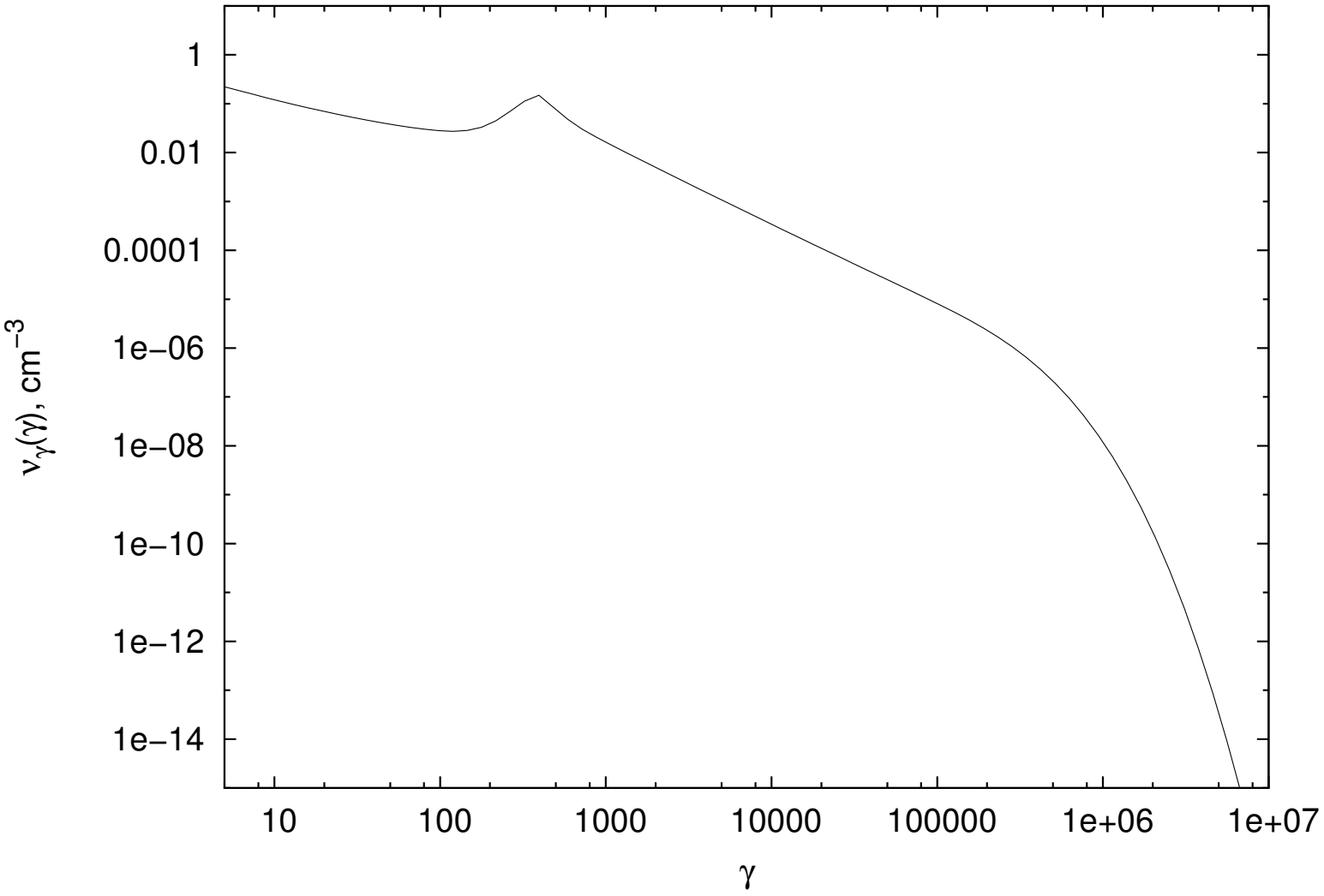

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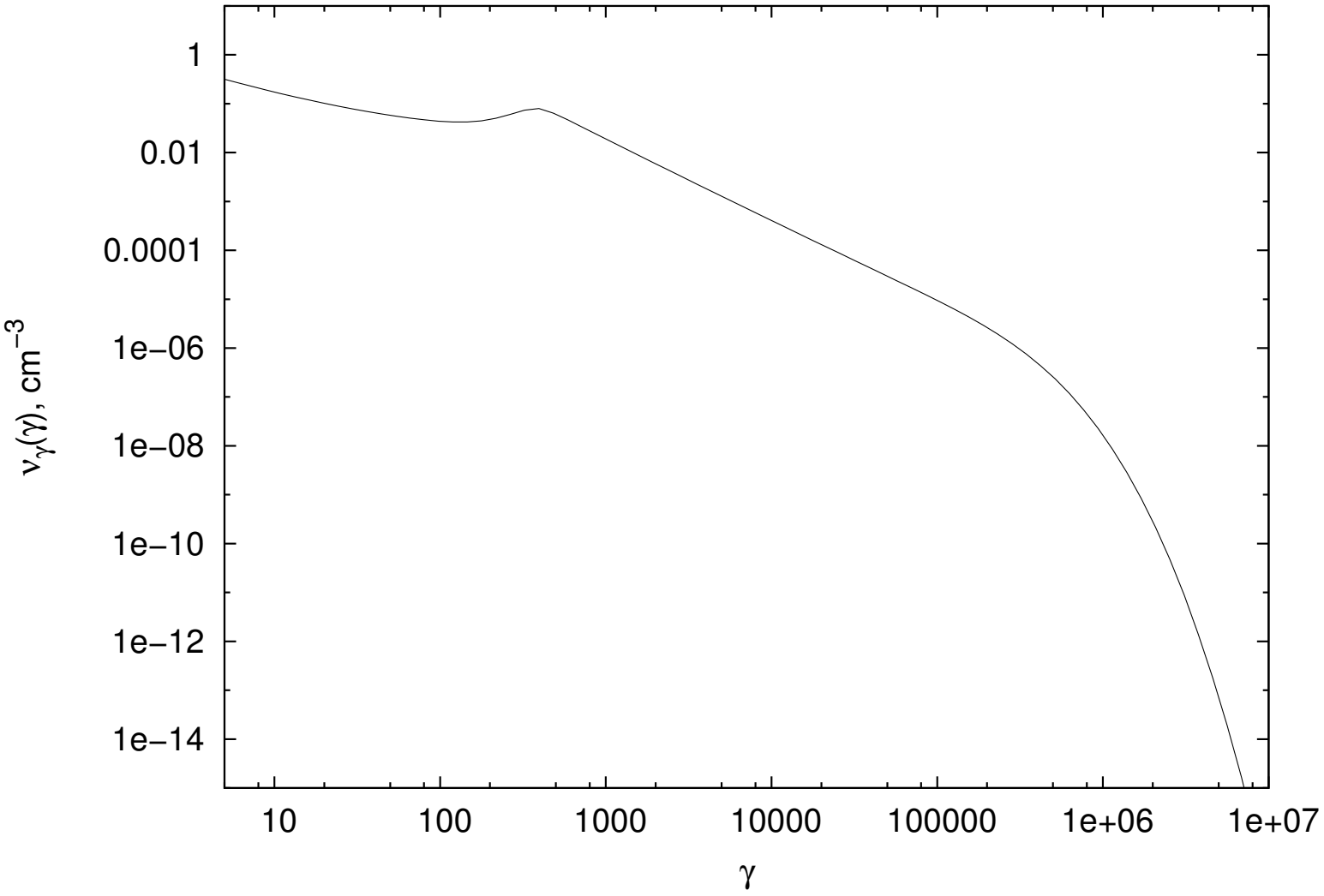

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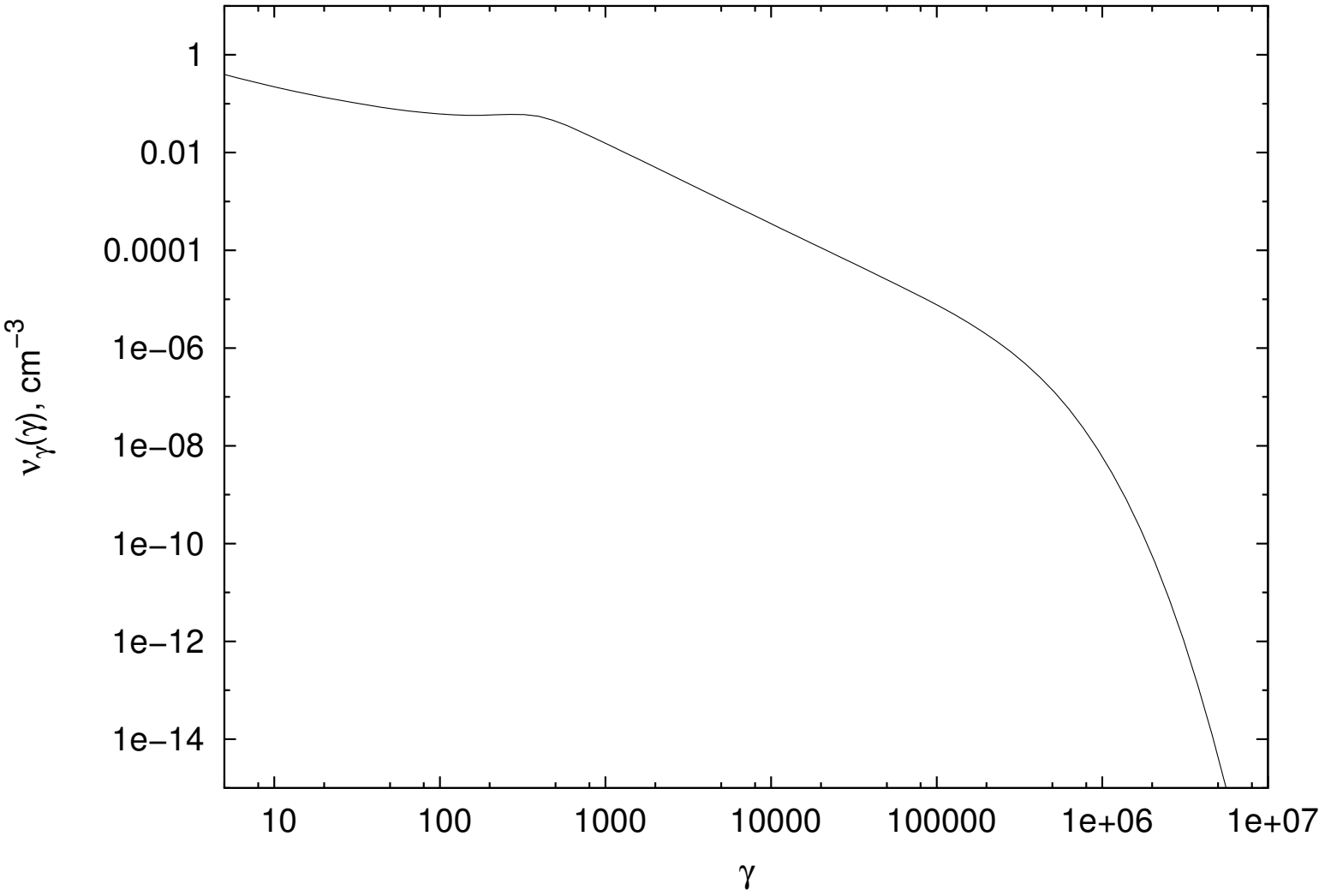

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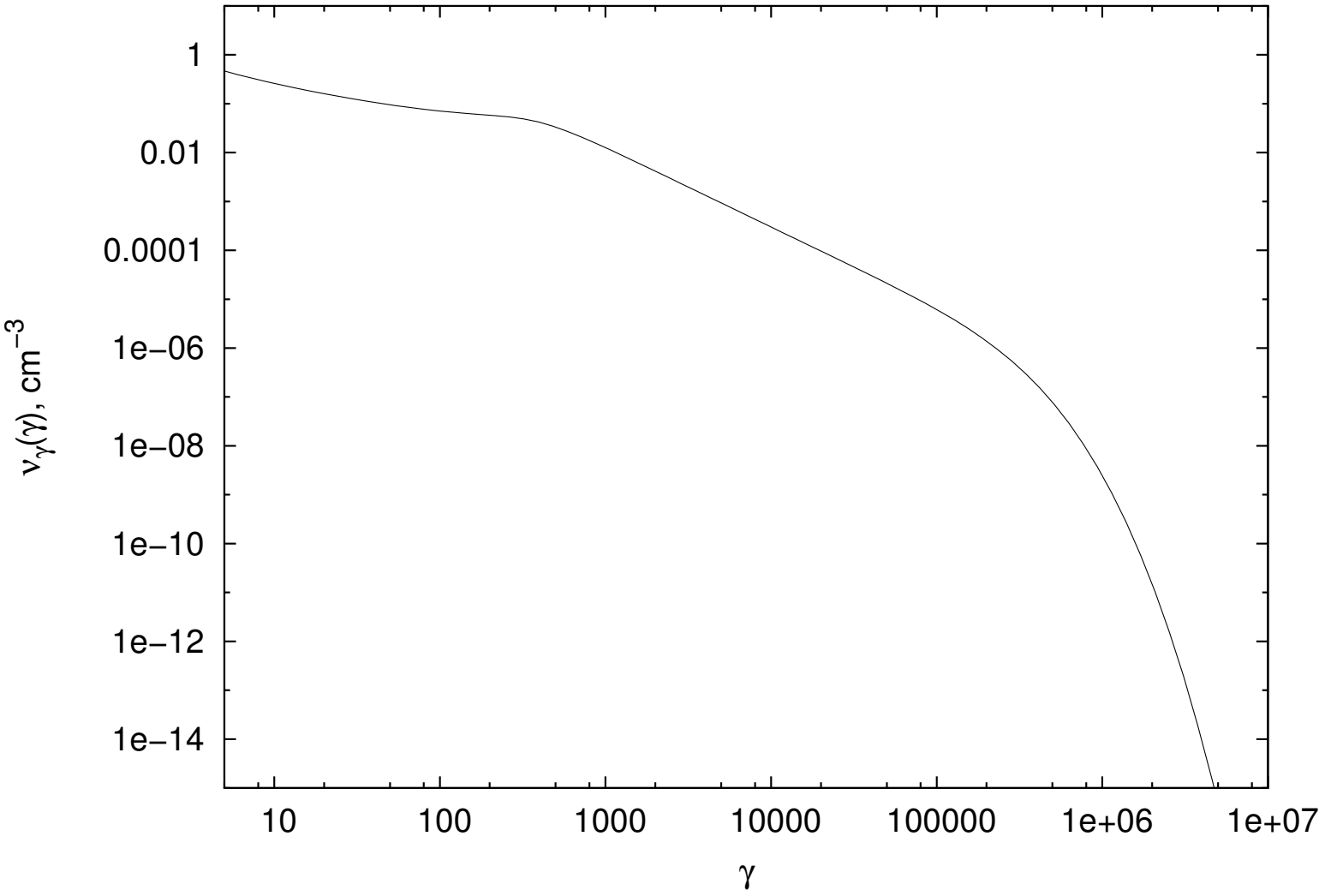

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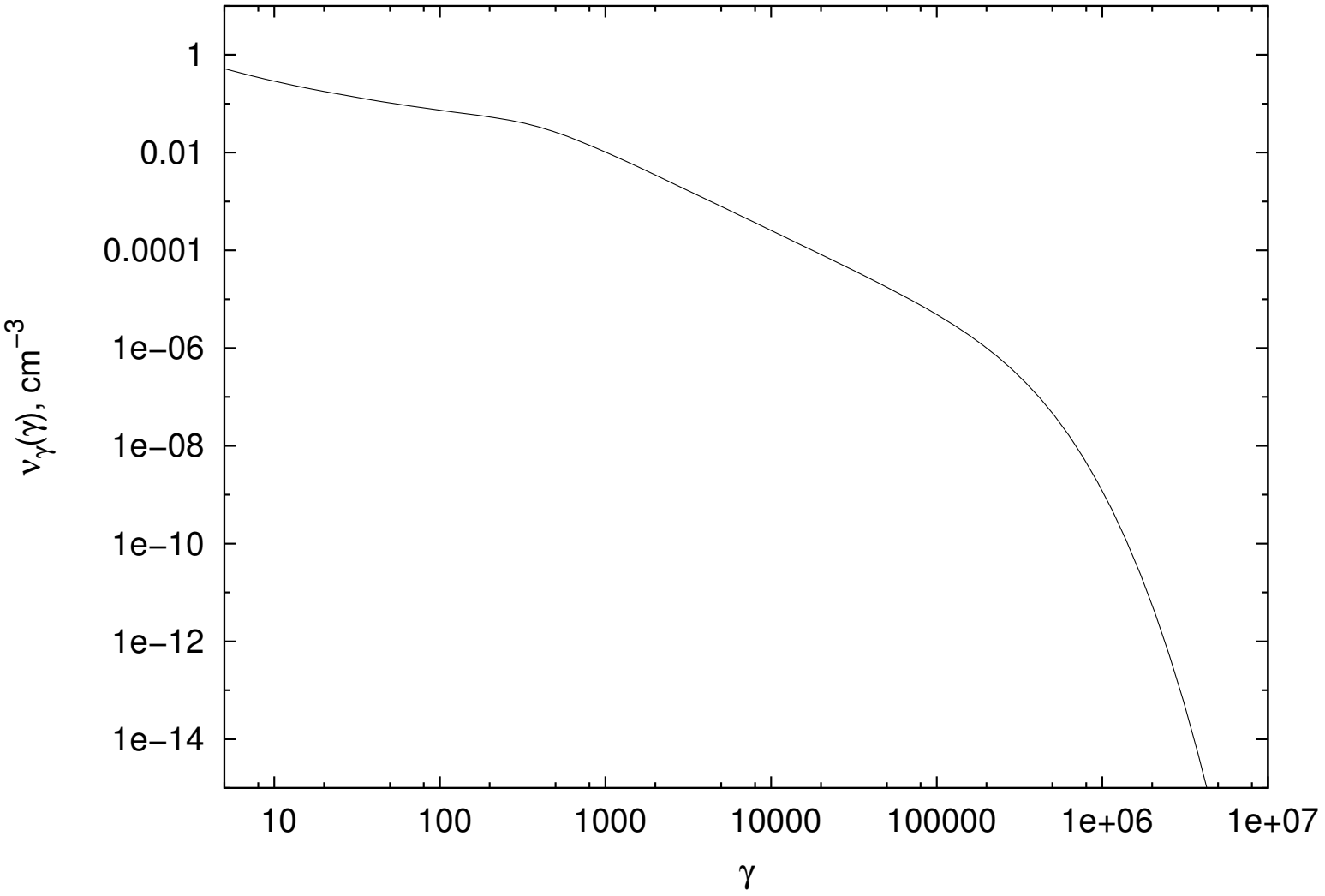

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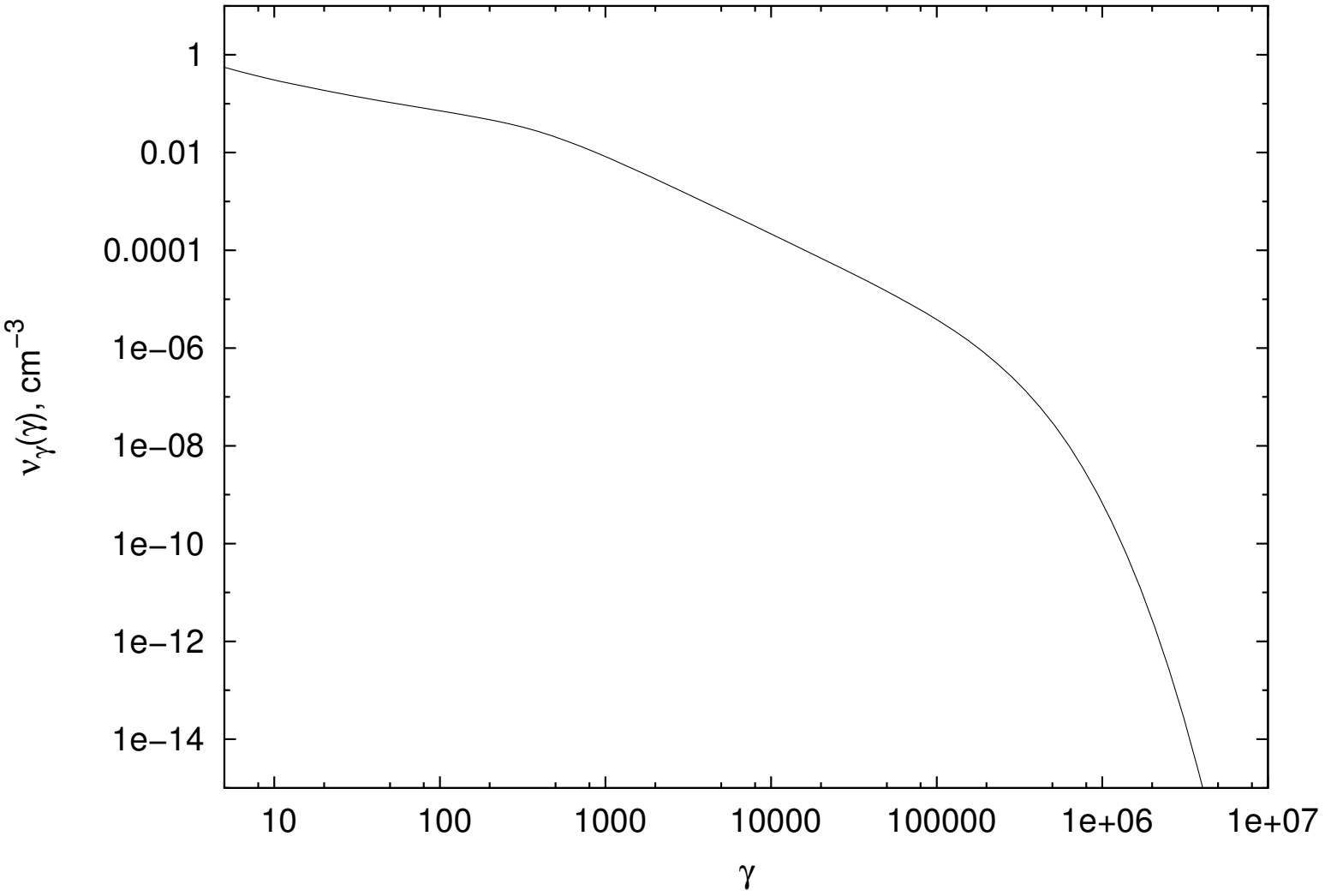

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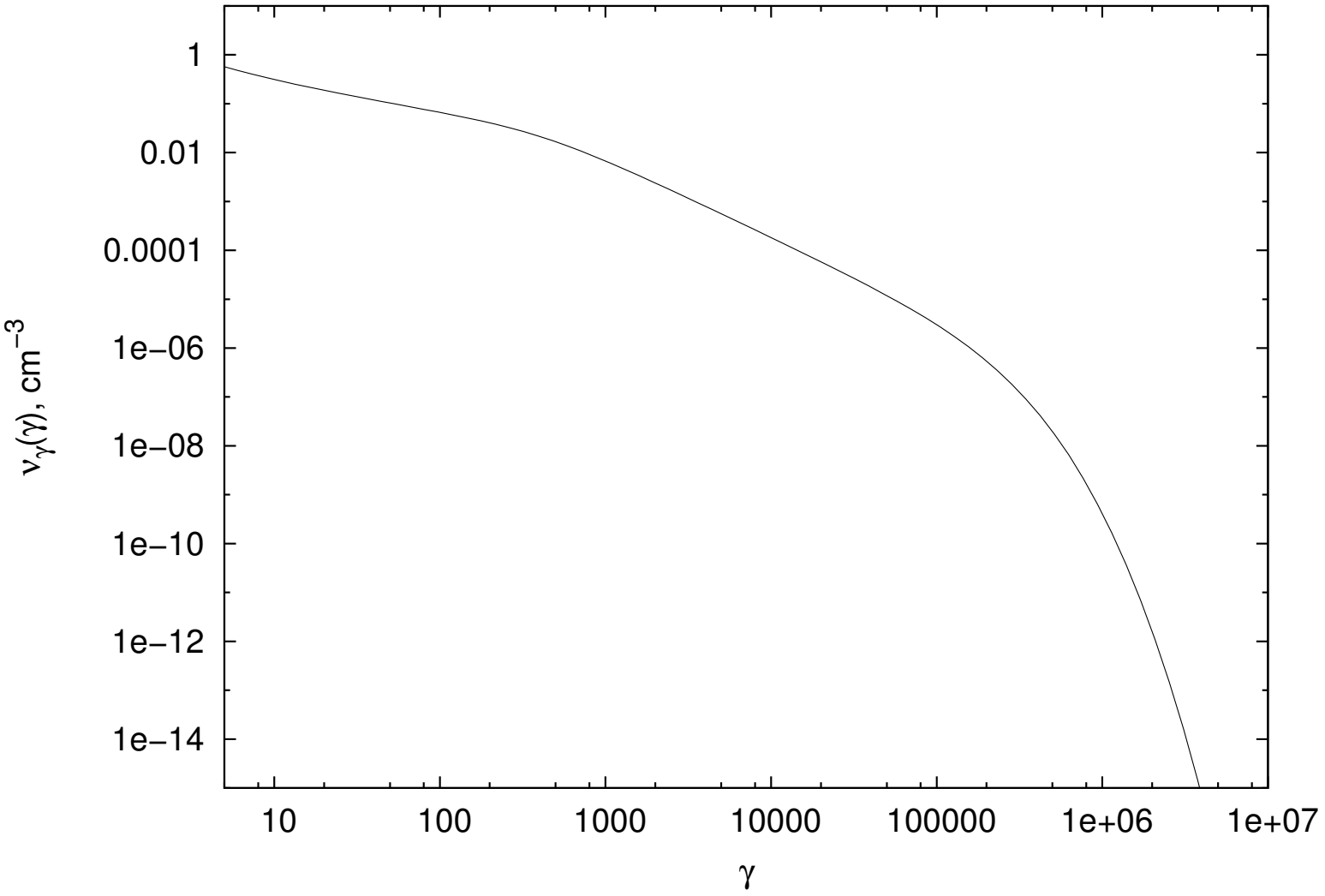

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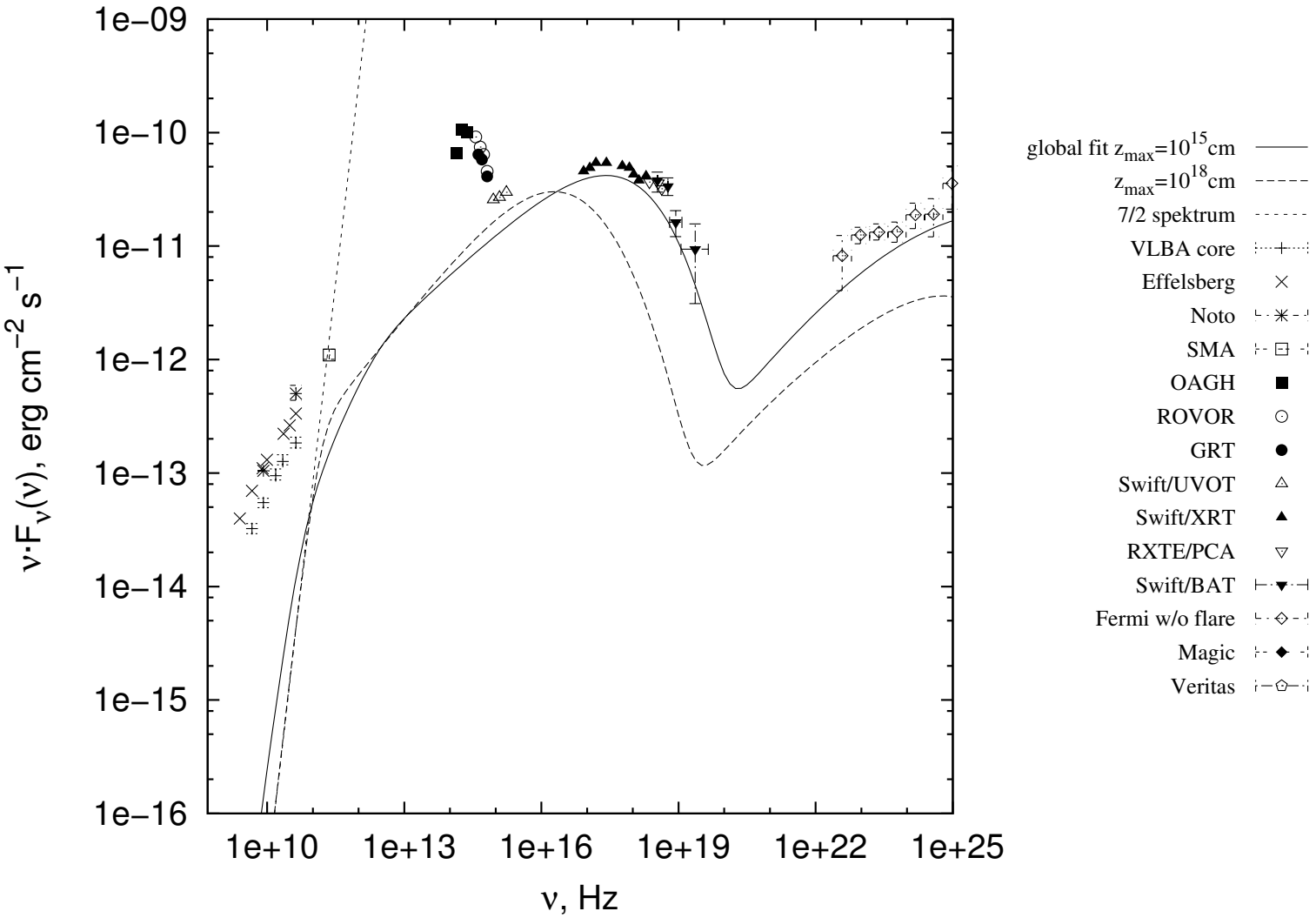
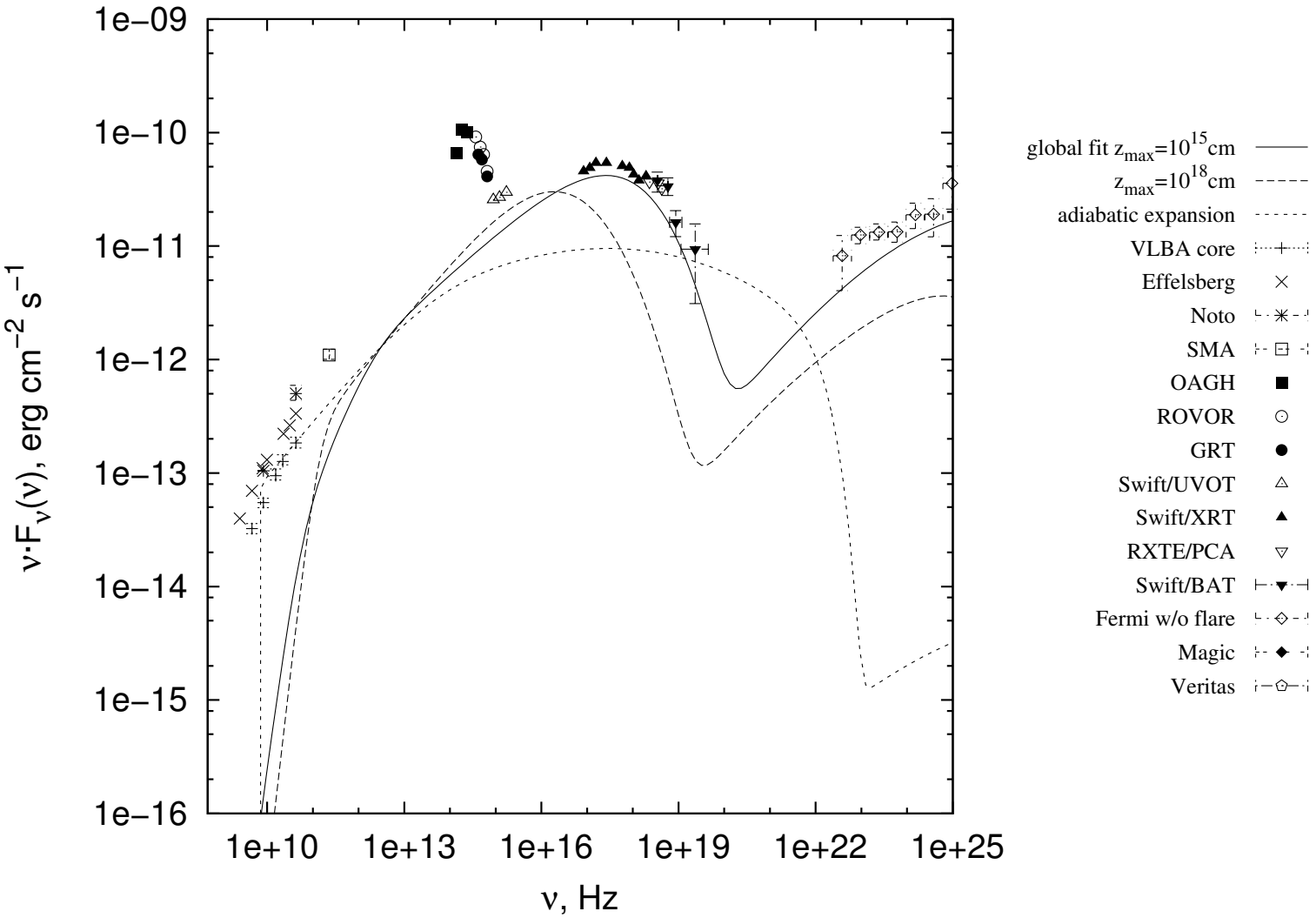


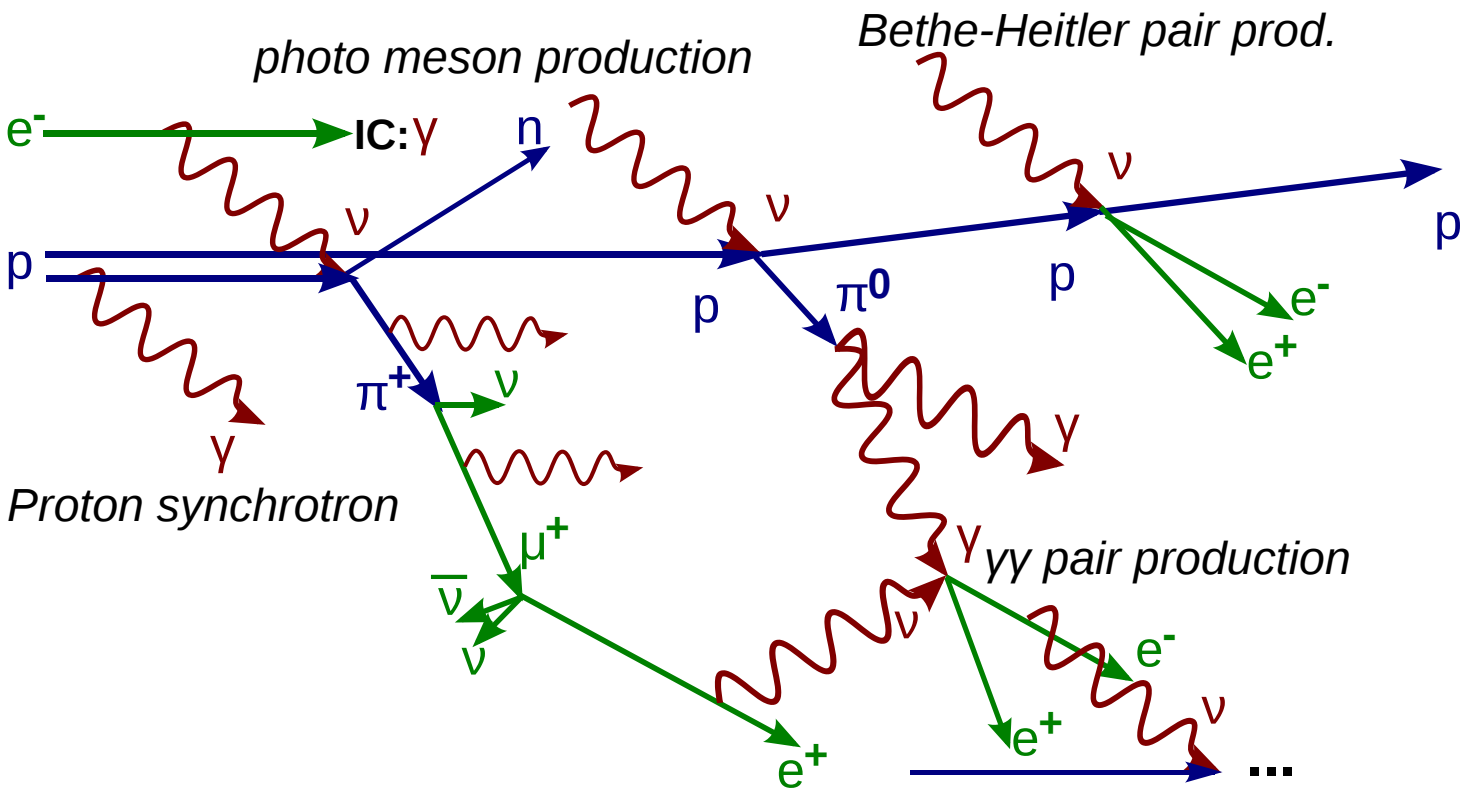
Effect of significant larger simulation region.

Adiabatic expansion



Effect of additional adiabatic expansion.

Outlook - hadronic processes



schematic of the dominant photohadronic processes [from: Weidinger 2012]

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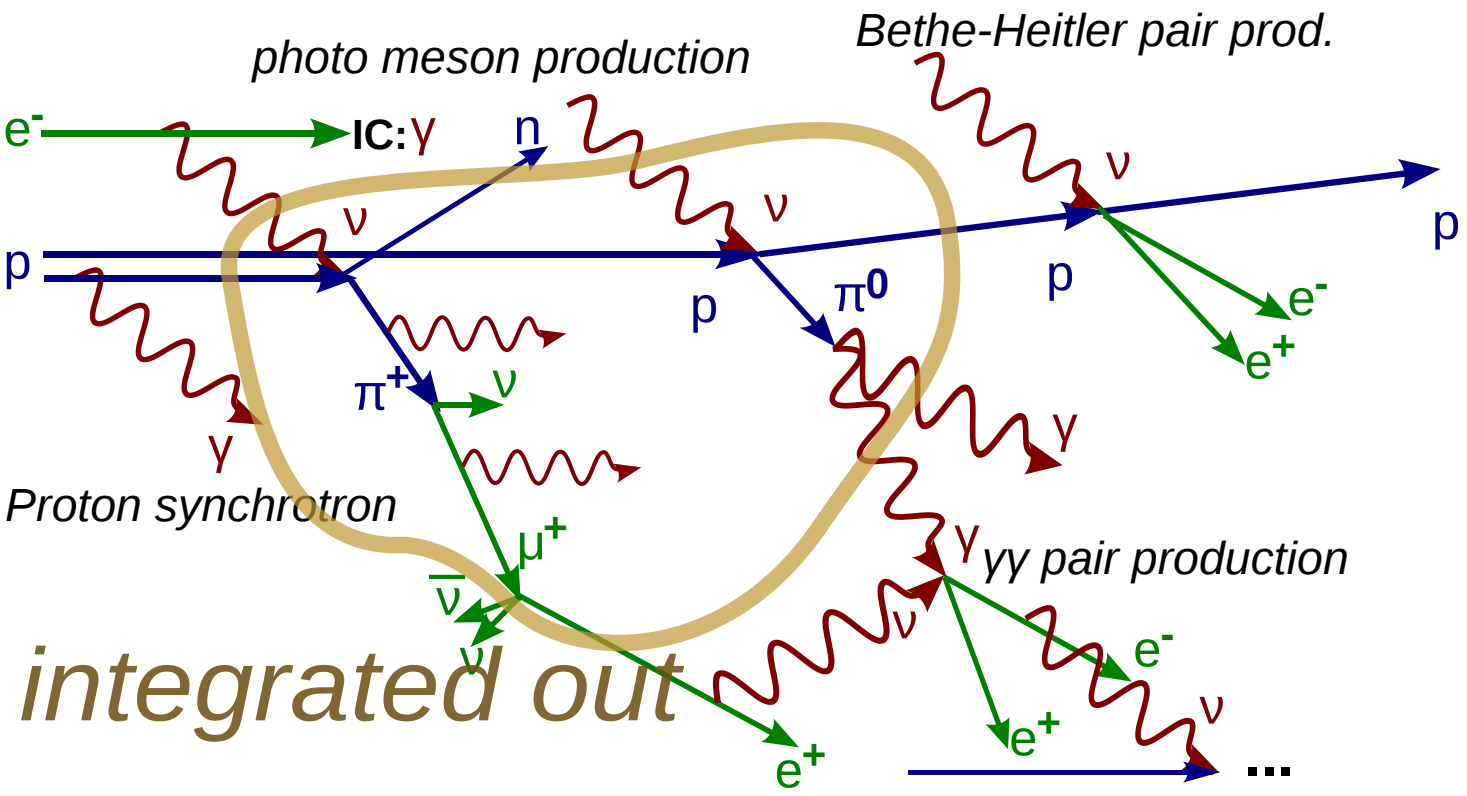

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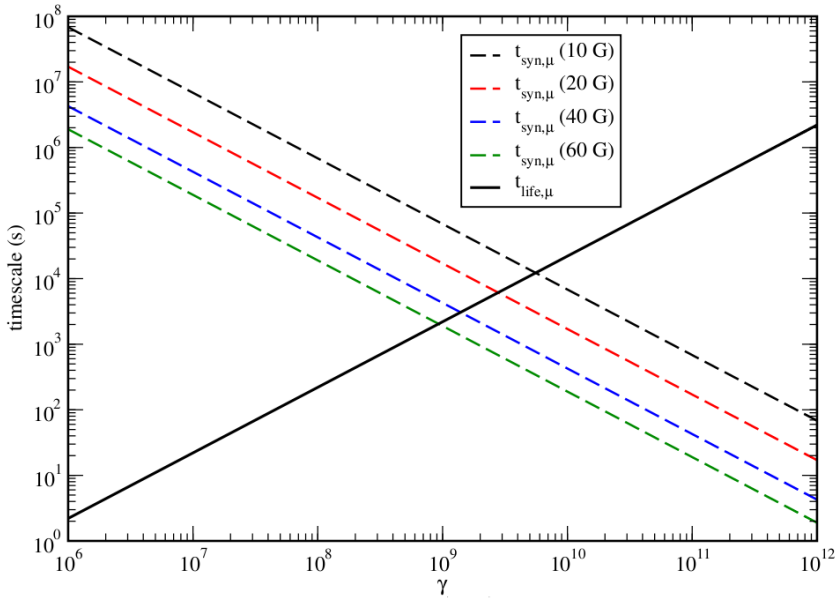
Hadronic processes

- cooling (and possibly acceleration) of intermediates is relevant in AGNs
- ⇒ using model by Hümmer et al. 2010
- How does this influence resulting SEDs and variation timescales?
- calculation of realistic, flavour splitted neutrino spectra



Muon lifetime vs. dominant cooling process [from: Weidinger 2012]

Conclusion

- self consistent spatially resolved SSC model
- addition of photohadronics (in progress)
- although higher numerical costs, reasonable computation times thanks to GPGPU
- access to radio information via adiabatic expansion
- investigate cooling and reacceleration effects on hadronic modelling
- also: time variability, morphology (epecially magnetic fields)

spatial resolution brings new opportunities, but new problems, too

Thank you

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calculations over several orders of magnitudes in Lorentz factor γ and photon energy $\nu \Rightarrow$

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- choose N_γ, $i \in [0, N_\gamma - 1]$
- fraction of neighbouring grid points $\delta_\gamma = \frac{\gamma_{i+1}}{\gamma_i} = \left(\frac{\gamma_{max}}{\gamma_{min}}\right)^{\frac{1}{N_\gamma - 1}}$
- get grid points via $\gamma_i = \gamma_{min} \cdot \delta_\gamma^i$
- cell interfaces $\gamma_i^{interface} = \gamma_{i-1/2} = \gamma_{min} \cdot \delta_\gamma^{(i-\frac{1}{2})} = \sqrt{\gamma_{i-1} \cdot \gamma_i}$
- cell size $\gamma_i^{interval\ size} = \gamma_{i+1}^{interface} - \gamma_i^{interface}$

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integrations are performed via second order *simpson's rule*, generalised for log-grids